Compton Camera Imaging

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Overview

- Compton camera imaging
  - in astronomy, nuclear medicine, and lately in homeland security screening applications.
- Cone (or Compton) transform
  - integrates a function (source intensity distribution) over conical surfaces.
- An inversion formula for the cone transform
  - through a relation between cone and Radon transforms.

Introduction

Conventional gamma cameras used in emission tomography determine the direction of an incoming γ-photon by "collimating" the detector (see Fig. 1).

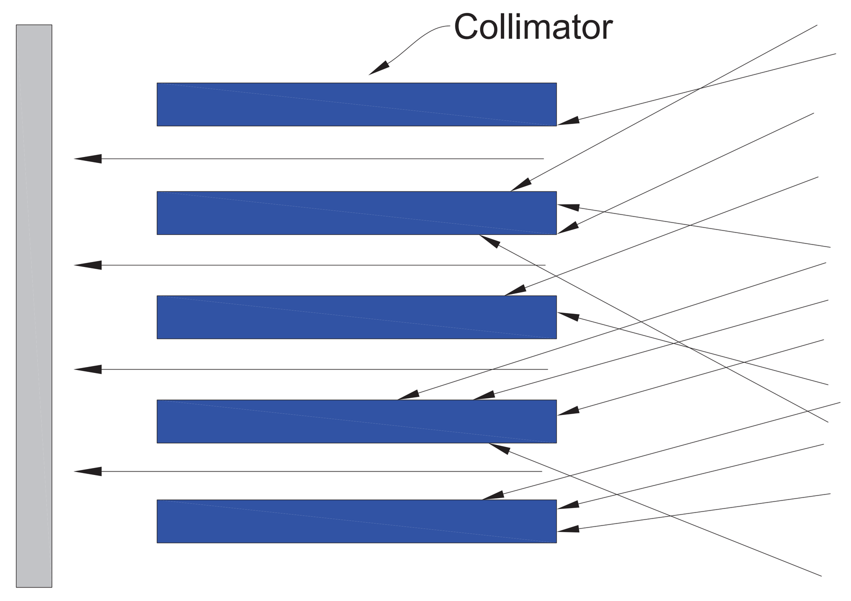

Figure 1: Collimation.

This technique leads to very low efficiency because only a small fraction of the radiation is transmitted through the collimator [2]. Thus, the acquired signal is weak and statistically noisy. The situation is similar in astronomy and is even more drastic for homeland security screening applications [1].

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Compton Camera

- Already used in astronomy as a telescope to detect atmospheric or cosmic γ-ray sources
- Locate the source by Compton scattering principle
- No mechanical collimation
- Dramatic increase in sensitivity
- Simultaneous multiple views of the object
- Two parallel detectors recording the position and energy of the incoming photon.
  - Compton scattering at the 1st detector,
  - Absorption at the 2nd detector.

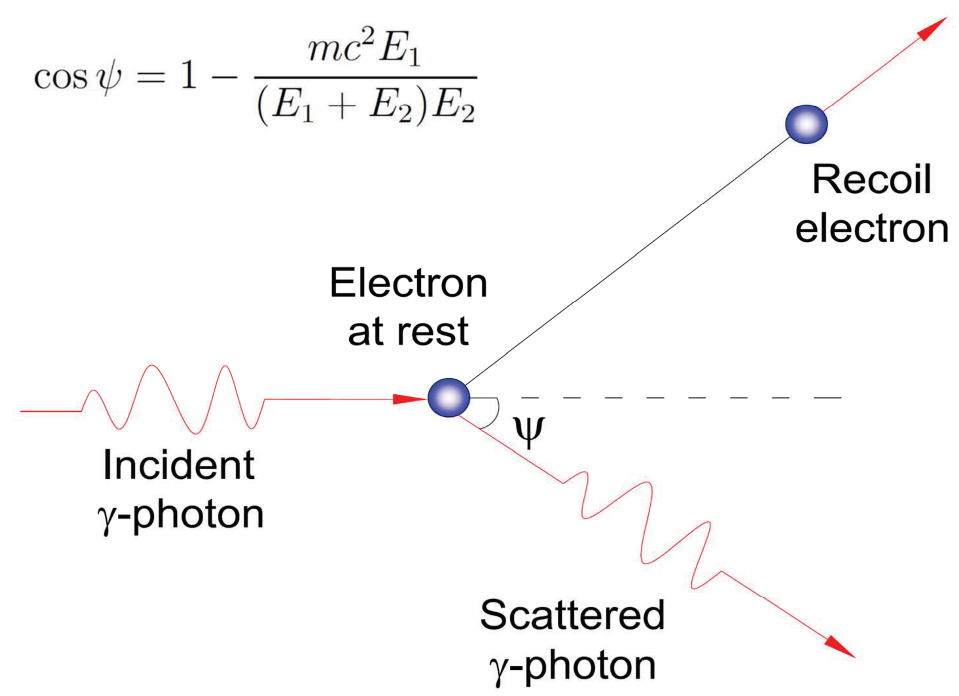

Figure 2: Compton Scattering.

- From the knowledge of β and the scattering angle ψ, we conclude that the photon originated from the surface of the cone with central axis direction β, vertex x_1 and opening angle ψ .

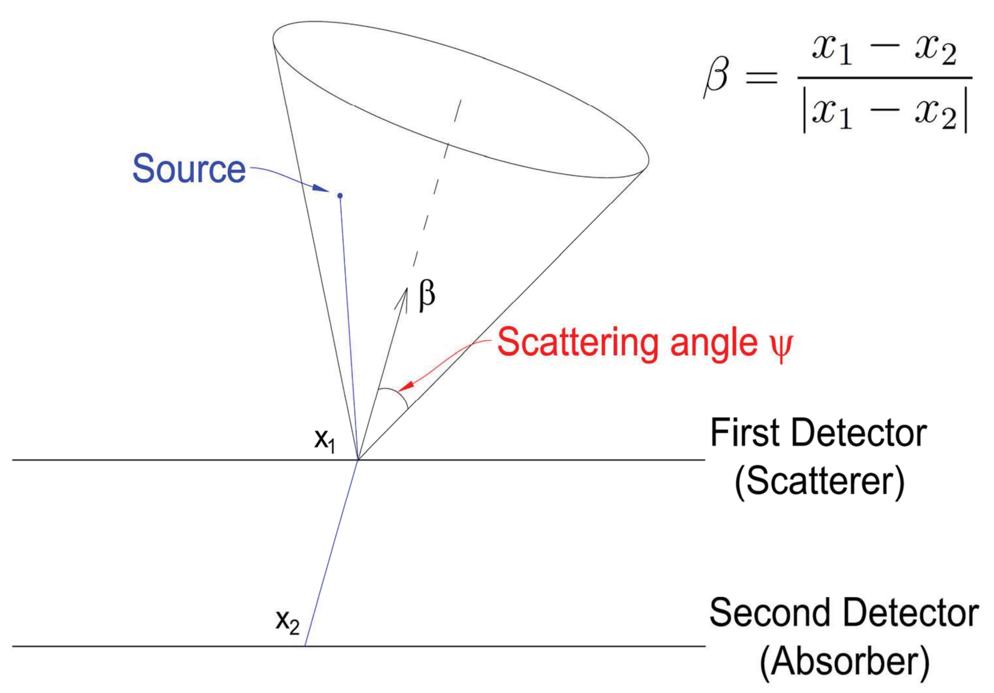

Figure 3: Compton Camera.

Mathematical Model

- $f(x)$-source intensity distribution function
- Compton camera data:

  Integrals of f over conical surfaces having vertex at the detector.
- The *cone* or *Compton transform*:

  $f(x) \longmapsto$ Integral of f over conical surfaces having vertex at the detector.
- Compton camera imaging

  GOAL: Recover source distribution f.

Cone Transform

- Equation of a cone in $\mathbb{R}^n$:

$$(x-u)\cdot\beta = |x-u|\cos\psi.$$

- Parametrization:

$$(u,\beta,\psi) \in \mathbb{R}^n \times S^{n-1} \times [0,\pi].$$

- n-dimensional *cone* transform of $f \in \mathcal{S}(\mathbb{R}^n)$:

$$Cf(u,\beta,\psi) = \int\limits_{(x-u)\cdot\beta = |x-u|\cos\psi} f(x)dx$$

dx-the surface measure on the cone.

Inversion of the Cone Transform

Theorem ([3]): Let $f \in \mathcal{S}(\mathbb{R}^n)$, $u \in \mathbb{R}^n$ and $\omega \in S^{n-1}$. If $n = 3$,

$$Rf(\omega,\omega\cdot u) = \frac{1}{2\pi^2}\int\limits_{S^2}\int\limits_0^{\pi} Cf(u,\beta,\psi)\sin\psi d\psi d\beta - \frac{\Delta_S(\Delta_S+2)}{4\pi^2}\left\{\int\limits_{S^2}\int\limits_0^{\pi} Cf(u,\beta,\psi)\log\frac{1}{|\omega\cdot\beta|}\sin\psi d\psi d\beta\right\}$$

and, if $n = 2$,

$$Rf(\omega,\omega\cdot u) = \frac{\Delta_S+1}{2}\int_0^{\pi} Cf(u,\omega^{\perp},\psi)\sin\psi d\psi.$$

Here, Δ_S is the Beltrami-Laplace operator on S^2 acting on ω, and R denotes the Radon transform whose inversion is well-known.

Reconstructions (2D)

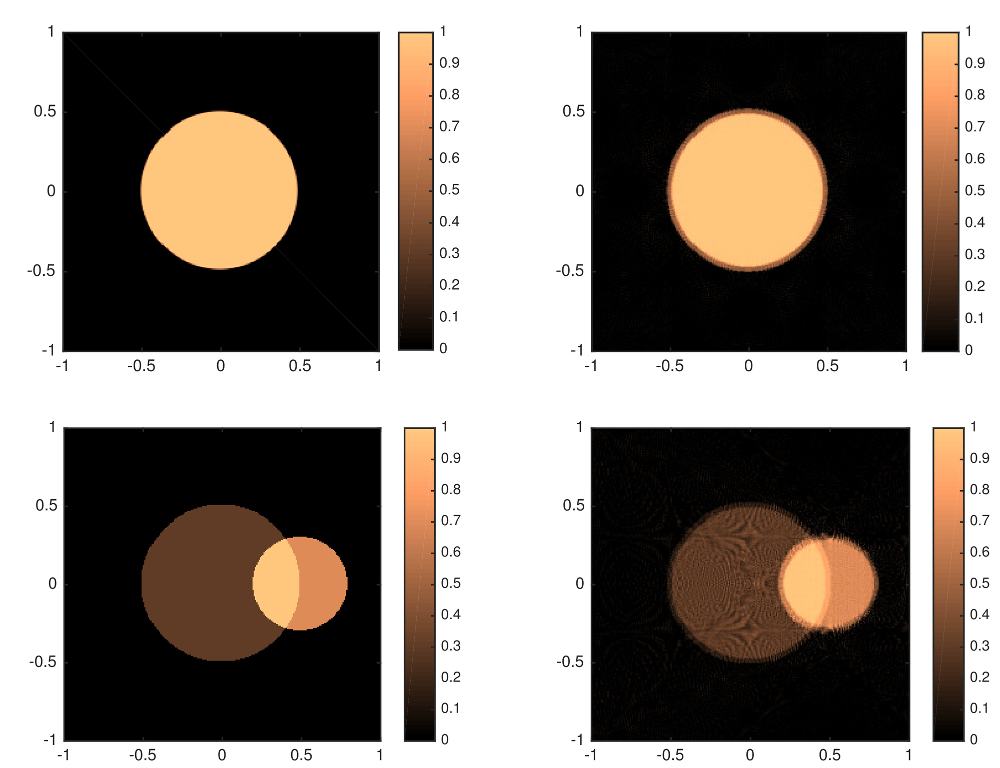

Figure 4: Phantom Reconstruction

256x256 image reconstructed from the simulated Compton data using 257 detectors per side and 200 counts for β and ψ each.

References

1. Allmaras M, Darrow D P, Hristova Y, Kanschat G and Kuchment P 2013 Detecting small low emission radiating sources *Inverse Problems Imaging* **7** 47-79
2. Cree M J and Bones P J 1994 Towards direct reconstruction from a gamma camera based on Compton scattering *IEEE Trans. Med. Imaging* **13** 398-409
3. Terzioglu F 2015 Some Inversion Formulas for the Cone Transform *Inverse Problems* **31** 115010

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