238U + 238U Collisions

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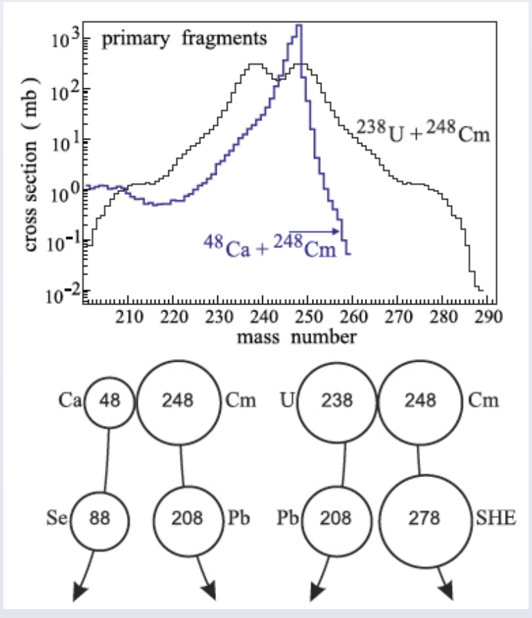
SUMMARY

We simulate the collision of two Uranium 238 nuclei using the time dependent superfluid local density approximation (TDSLDA). We test out four various initial relative orientations (shown below), while varying the initial boost energies of the nuclei as well as setting the relative phases of the paring gaps to be in or out of phase. After completing the simulations we then calculate various observables. Note, all calculations were performed at zero impact parameter, but will eventually be extended to finite impact parameters.

Motivation:

A big motivation for the collision of heavy elements is to produce super-heavy elements (SHEs) and neutron rich nuclei close to the neutron drip line via multi-nucleon transfer (MNT) reactions.

As evidence, examine the following figure



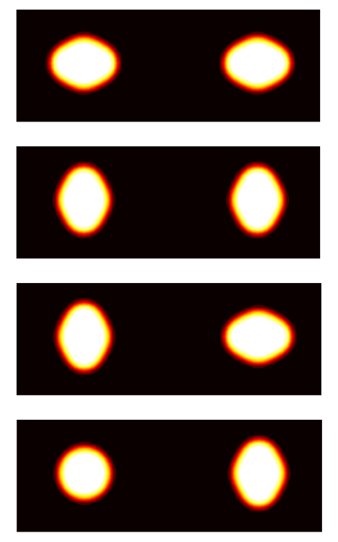
Acquired from source [1].

which shows that the probability of producing super heavy nuclei is far greater in the case of colliding two heavy nuclei when compared to colliding one heavy and one light nuclei.

In our project we will also focus on extending traditional approaches such as TDHF or TDHF+BCS to TDSLDA to see if pairing plays a role in these types of collisions.

Orientations:

From top to bottom the orientations are labeled XX, YY, YX, ZY, where X,Y,Z denote the largest moment of inertia of each colliding nuclei along the collision axis OX.



METHOD

Theoretical framework:

The quasiparticle wavefunctions

$$\phi_k(\vec{r}) = \begin{pmatrix} u_{k\uparrow}(\vec{r}) \\ u_{k\downarrow}(\vec{r}) \\ v_{k\uparrow}(\vec{r}) \\ v_{k\downarrow}(\vec{r}) \end{pmatrix}$$

satisfy the following evolution equations

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k\uparrow}(\vec{r},t) \\ u_{k\downarrow}(\vec{r},t) \\ v_{k\uparrow}(\vec{r},t) \\ v_{k\downarrow}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\vec{r},t) & h_{\uparrow\downarrow}(\vec{r},t) & 0 & \Delta(\vec{r},t) \\ h_{\downarrow\uparrow}(\vec{r},t) & h_{\downarrow\downarrow}(\vec{r},t) & -\Delta(\vec{r},t) & 0 \\ 0 & -\Delta^*(\vec{r},t) & -h^*_{\uparrow\uparrow}(\vec{r},t) & -h^*_{\uparrow\downarrow}(\vec{r},t) \\ \Delta^*(\vec{r},t) & 0 & -h^*_{\downarrow\uparrow}(\vec{r},t) & -h^*_{\downarrow\downarrow}(\vec{r},t) \end{pmatrix}\begin{pmatrix} u_{k\uparrow}(\vec{r},t) \\ u_{k\downarrow}(\vec{r},t) \\ v_{k\uparrow}(\vec{r},t) \\ v_{k\downarrow}(\vec{r},t) \end{pmatrix}$$

for any local energy density functional (EDF). Here we focused on the SeaLL1 NEDF [2].

These solutions can then be used to construct various densities,

$$n(\vec{r}) = \sum_{k,s} v^*_{k,s}(\vec{r})v_{k,s}(\vec{r}),$$

$$\kappa(\vec{r}) = \sum_k v^*_{k\uparrow}(\vec{r})u_{k\downarrow}(\vec{r}),$$

$$\tau(\vec{r}) = \sum_{k,s} \vec{\nabla}v^*_{k,s}(\vec{r})\cdot\vec{\nabla}v_{k,s}(\vec{r}),$$

$$\vec{s}(\vec{r}) = \sum_{k,s,s'} \vec{\sigma}_{s,s'}v^*_{k,s}(\vec{r})v_{k,s'}(\vec{r}),$$

$$\vec{J}(\vec{r}) = \tfrac{1}{2i}(\vec{\nabla}-\vec{\nabla}')\times\vec{s}(\vec{r},\vec{r}')|_{\vec{r}=\vec{r}'}$$

$$\vec{j}(\vec{r}) = \tfrac{1}{2i}\sum_{k,s}\{v_{k,s}(\vec{r})\vec{\nabla}v^*_{k,s}(\vec{r}) - v^*_{k,s}(\vec{r})\vec{\nabla}v_{k,s}(\vec{r})\}$$

which in turn are used to calculate the energy density functional, various observables, such as kinetic energies, quadrupole and octupole moments, etc...

Boosts:
To perform boosts we apply the following transformation,

$$\begin{pmatrix} u_{k\sigma}(\vec{r},t) \\ v_{k\sigma}(\vec{r},t) \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix}\begin{pmatrix} u_{k\sigma}(\vec{r},t) \\ v_{k\sigma}(\vec{r},t) \end{pmatrix}$$

where,

$$\chi = \pm\frac{p_x x}{\hbar} \quad p_x = \sqrt{\frac{m(e_{cm}-e_{coul})}{A}} \quad \sigma = \uparrow,\downarrow$$

Above χ is positive if we are in the left half of the box, and negative if we are in the right side of the box, with a smooth transition between the two near the center and outer boundaries.

The simulations were run on Oak Ridge's Summit supercomputer.

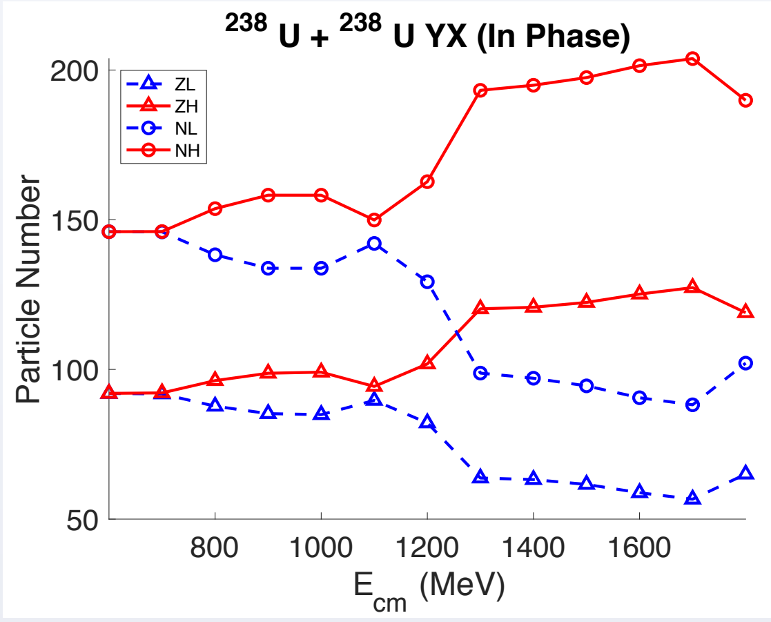
Space discretization:

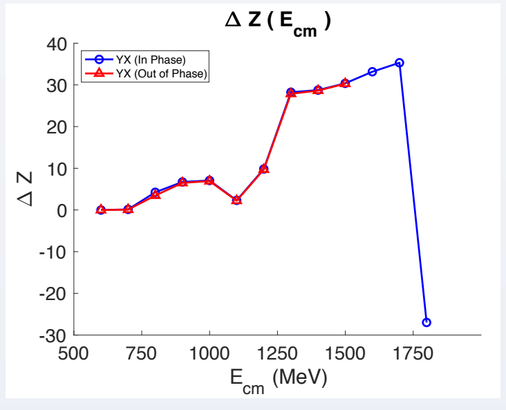
Δx = Δy = Δz = 1.25,
Lx = Ly = 30 fm , Lz = 80 fm,
# of PDES = 16 Nx Ny Nz = 589,824,
Δt = 0.03 fm/c,
GPUs per run = 720.

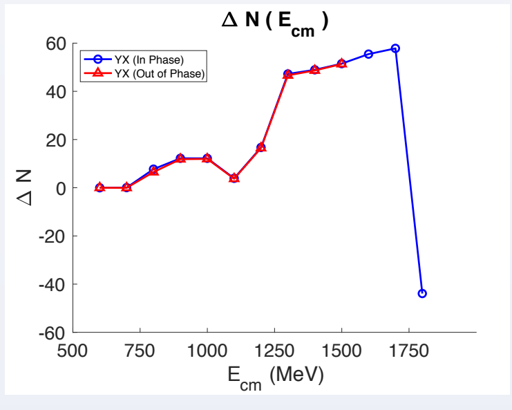
Nx = Ny = 24 , Nz = 64,
pcutoff = $\pi\hbar/\Delta x$ ~ 500 MeV/c,
# time steps per run ~ 30,000,
wall time per run ~ 3 hrs,

RESULTS

Multi-nucleon transfer in case of tip to waist collisions.

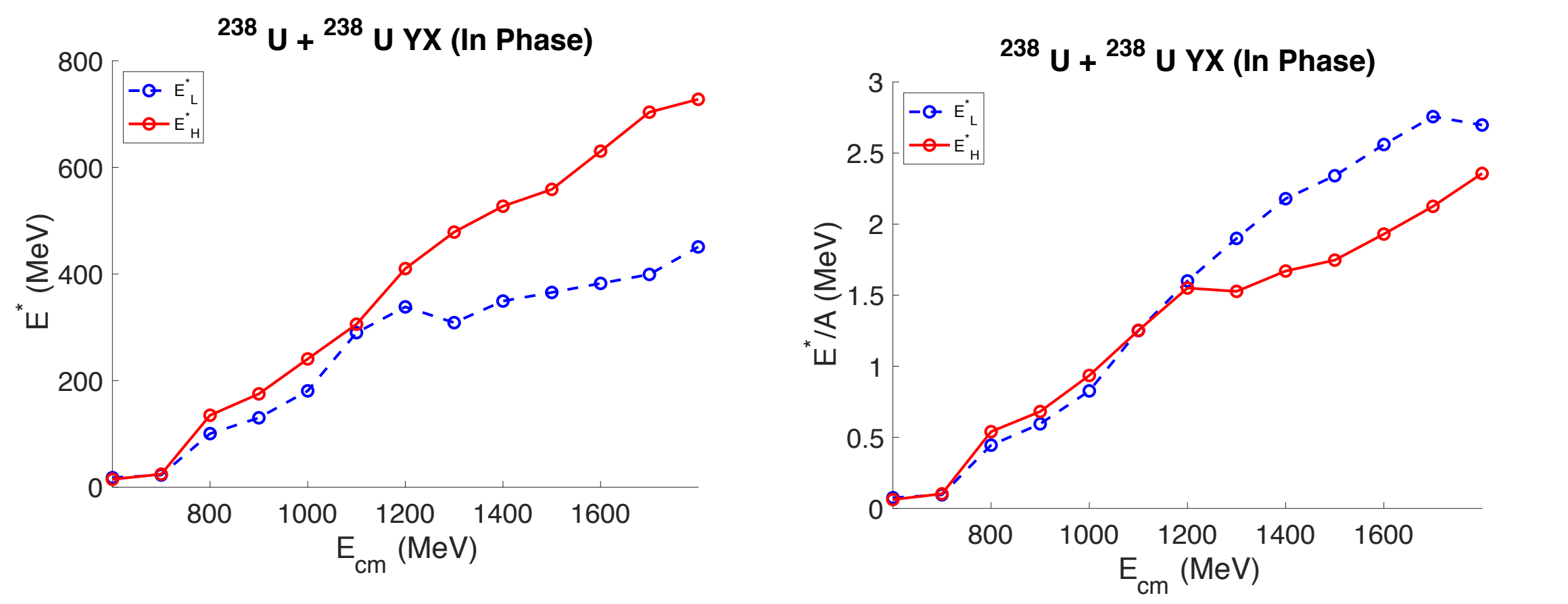






We plot the change in particle number for the left fragment. Notice the reversal in the direction of transfer between 1700 MeV and 1800 MeV.

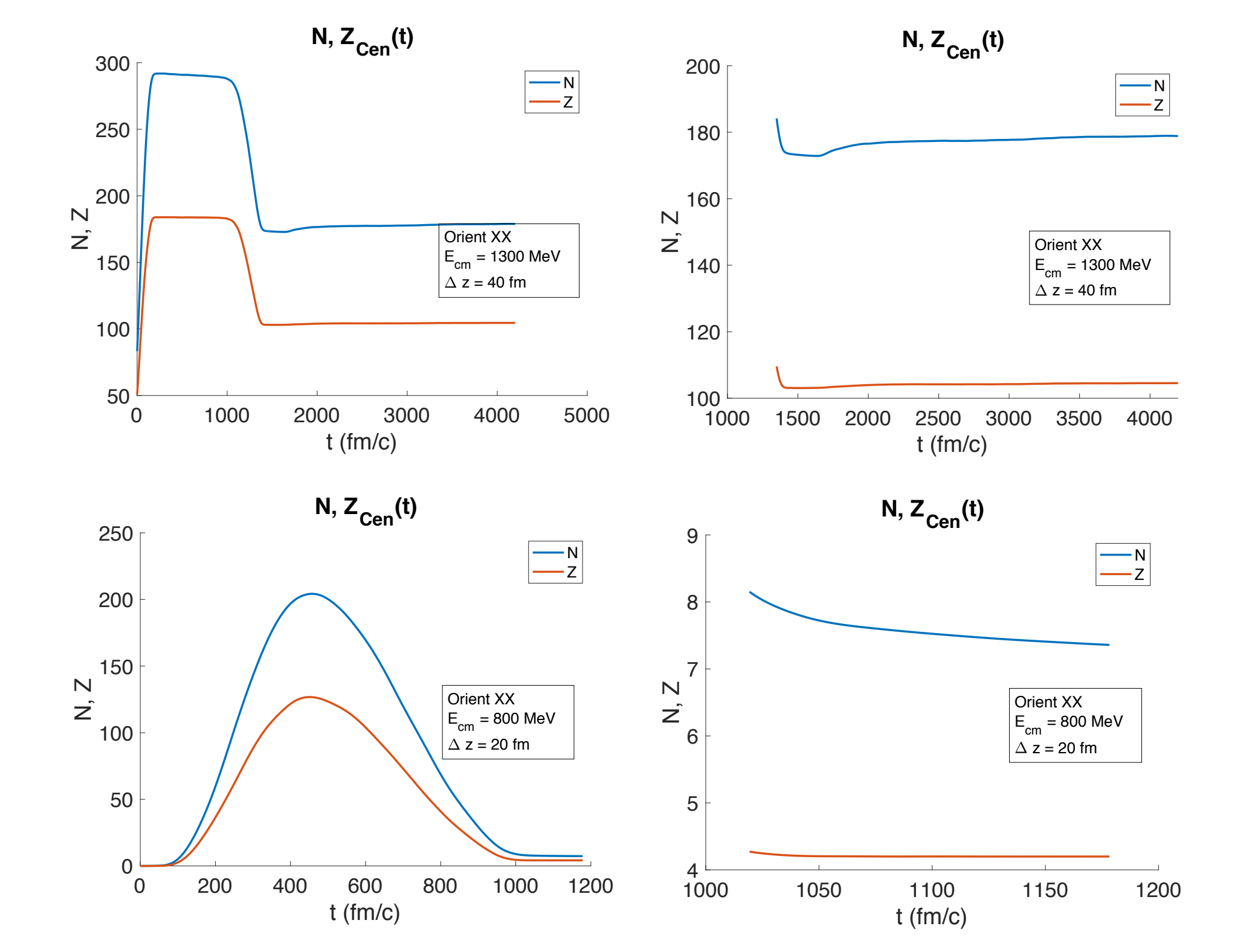
Excitation Energies:



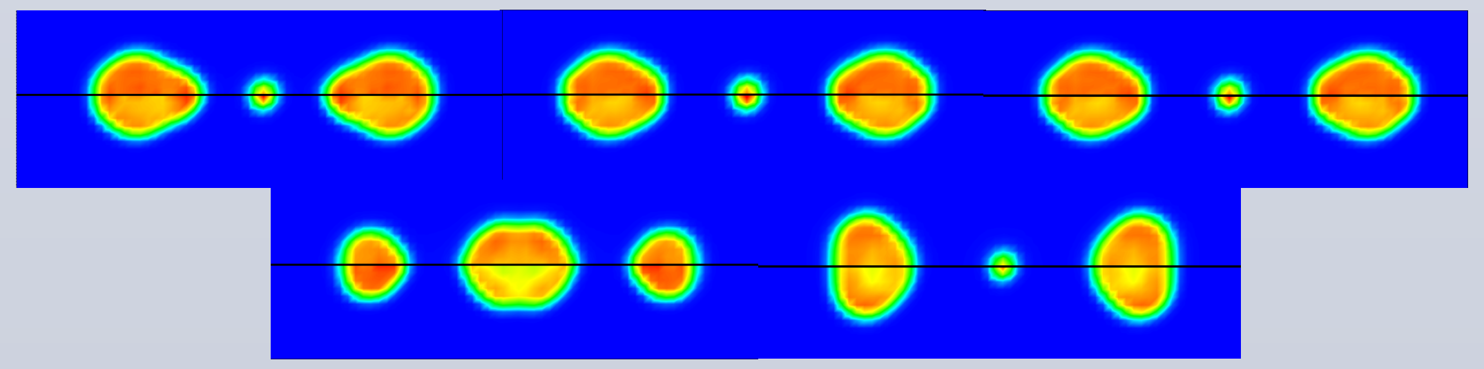
Ternary quasi fission:

We observed several collisions where a third fragment was formed.

Ternary Events						
Ecm (MeV)	DeltaZ (fm)	Orient	Phase	N	Z	A
750	20	XX	In phase	6.1	3.4	9.5
800	20	XX	In phase	7.4	4.2	11.6
800	20	XX	Out of phase	6.7	3.8	10.5
1300	40	XX	In phase	178.9	104.5	283.4
1400	20	YY	In phase	8.2	3.4	11.6

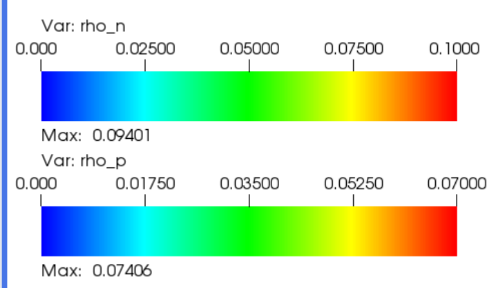


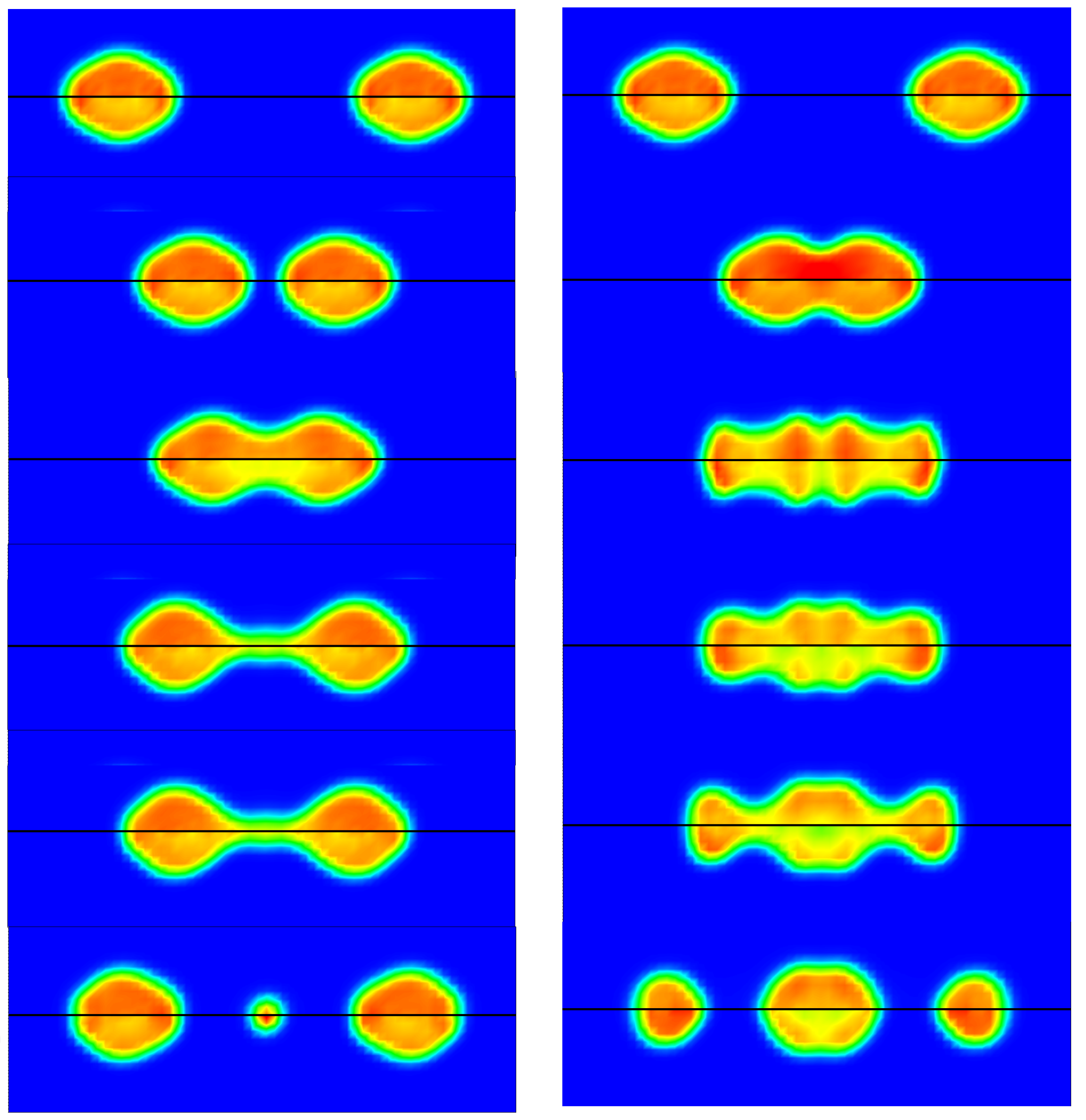
We typically observe small ternary fragments, but sometimes very large ones can form.



Examples:

Displayed are the number densities (upper half) and protons (lower half) as a function of time for runs at center of mass energies 800 MeV (left panel) and 1300 MeV (right panel) respectively, with the pairing field between the two nuclei in phase. In both cases the final time was 1080 fm/c.



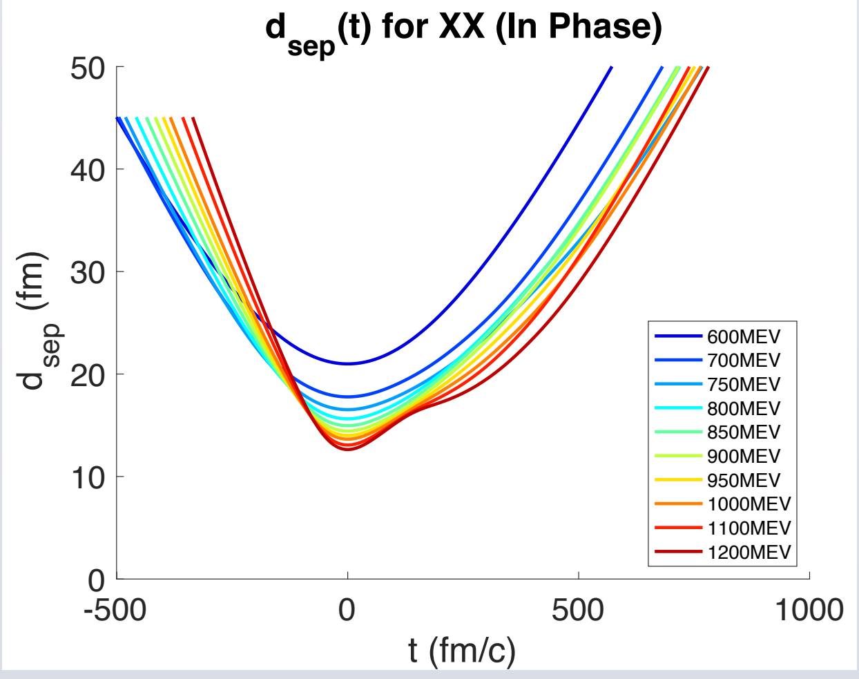


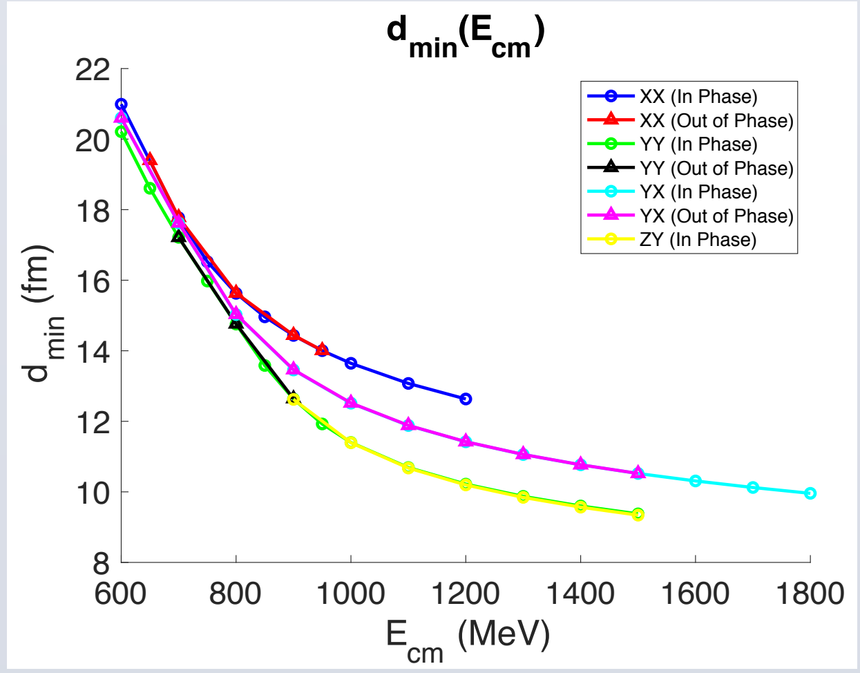
Other quantities:

The separation distance between the two fragments, given by,

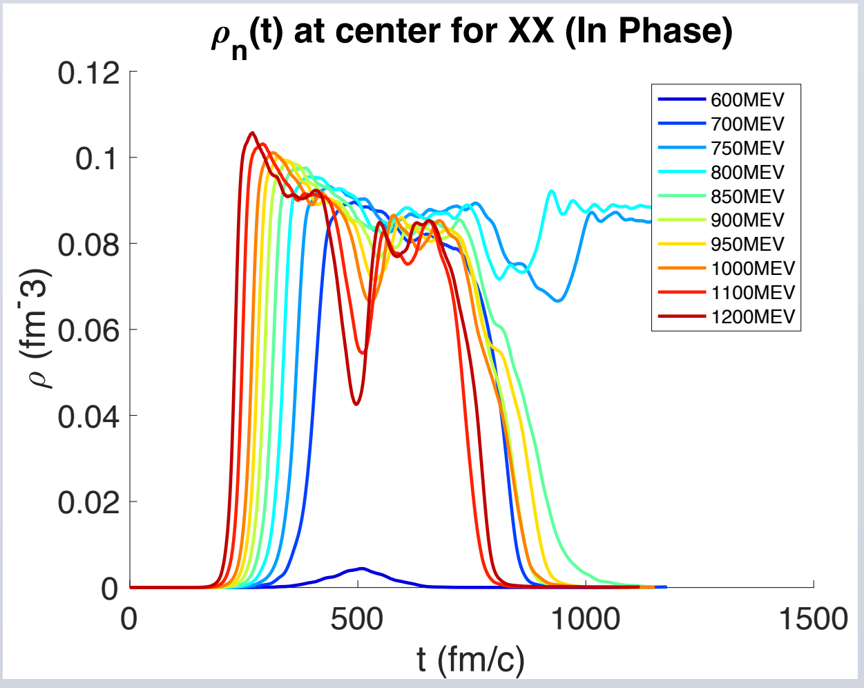
$$d_{sep} = \int_{V_R}\vec{r}\rho(\vec{r})d^3r - \int_{V_L}\vec{r}\rho(\vec{r})d^3r$$

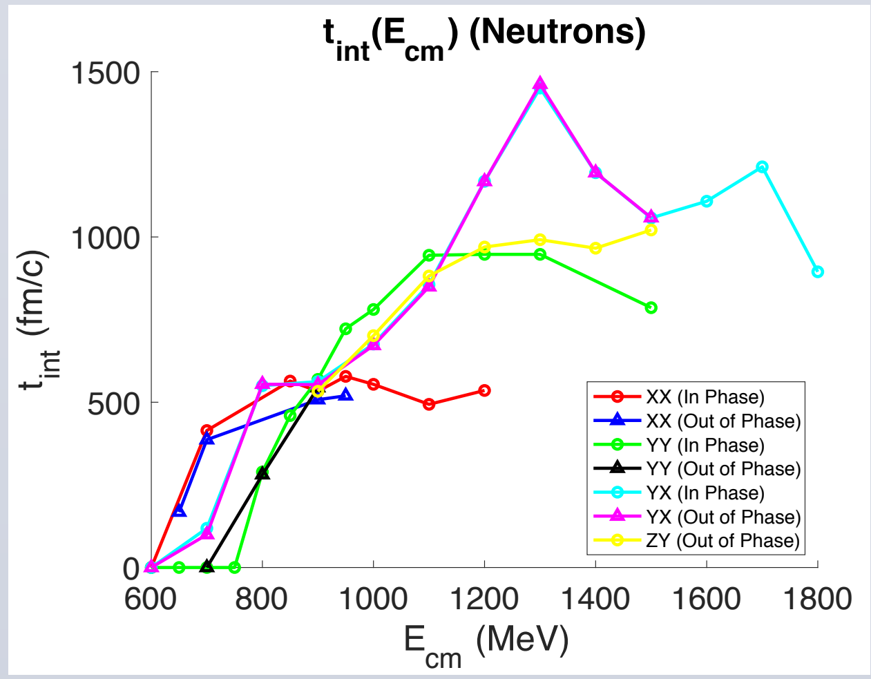
was tracked as a function of time for all runs, and used to extract the minimum separation between the two nuclei.





The interaction time is defined as the time interval when the neutron central density was larger 0.04 fm^{-3}. This prescription does not work for ternary events.



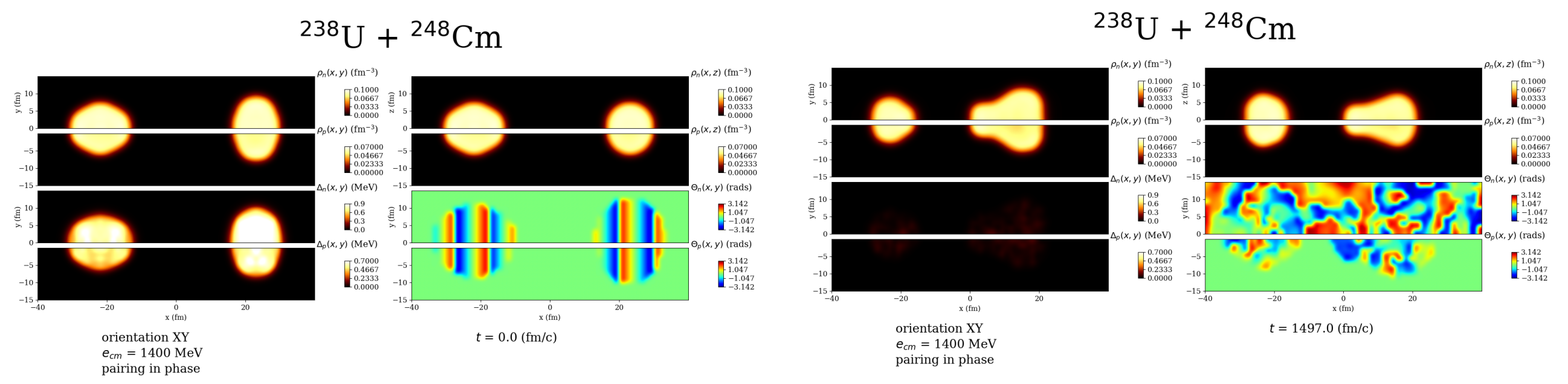


Additional quantities were also extracted, such as the energy loss, TKE before and after the collision, maximum density, and so forth, but are not included in this presentation.

FUTURE OUTLOOK

Other collisions:

We plan to examine the collisions ^{238}U+^{248}Cm and ^{232}Th+^{250}Cf.



Above is the before and after of a preliminary ^{238}U+^{248}Cm collision.

Mass widths of reaction products:

We will be applying the Balian Veneroni prescription for calculating the mass widths. This means we run our simulation until we arrive at our final state (in most cases two well separated nuclei at 50 fm). Then we apply the following transformation to the wavefunctions,

$$\begin{pmatrix} u_k(\vec{r},t) \\ v_k(\vec{r},t) \end{pmatrix} \rightarrow \begin{pmatrix} e^{\epsilon\hat{N}} & 0 \\ 0 & e^{-\epsilon\hat{N}} \end{pmatrix}\begin{pmatrix} u_k(\vec{r},t) \\ v_k(\vec{r},t) \end{pmatrix}$$

where ε is a small real number [3].

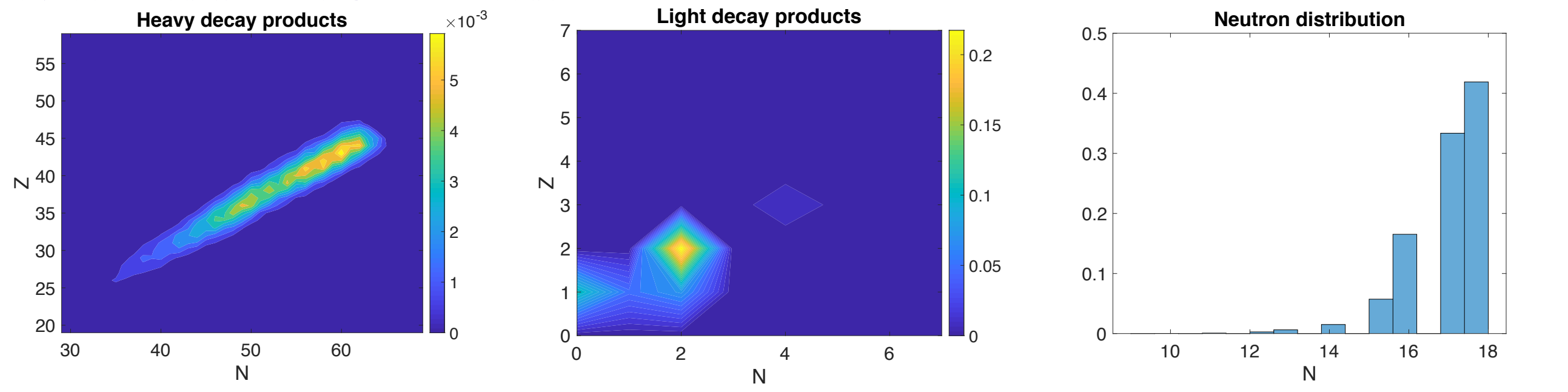
From here we run the solution backwards in time until we reach two nuclei separated by 45 fm (the initial separation distance). Using the initial and final densities we calculate the following observable,

$$\sigma^2 = \lim_{\epsilon\rightarrow 0}\frac{1}{2\epsilon^2}\,\mathrm{Tr}\,(\rho_0-\rho_\epsilon)^2$$

Post collision decay modes:

We will calculate the decay products for the excited nuclei after the collision, either by using the GEMNI code or another code.

Below is an example of how a result might look like for the decay of ^{238}U with excitation energy 200 MeV obtained with the GEMINI code



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