

(t,pf) reactions can lead to new insight into fission dynamics

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In 1956, A. Bohr suggested that low energy fission may be understood in terms of a very few levels in the transition nucleus. Although the level spacing in the compound nucleus at an excitation energy of about 6 MeV is of the order of 1 eV or less, most of the excitation energy goes into deformation energy during the passage from the initially excited nucleus to the highly deformed transition state nucleus (or saddle point). Hence, the transition state nucleus is thermodynamically “cold” and is expected to have a spectrum of excited states analogous to those of a normal nucleus near its ground state.

When the excitation energy of the compound nucleus is approximately equal to the fission barrier, present evidence strongly supports the concept of Bohr (1956) that fission occurs through one or only a few channels. Information on the properties of these transition levels is obtained from a study of fission-fragment angular distributions. Extensive discussion of fission-fragment

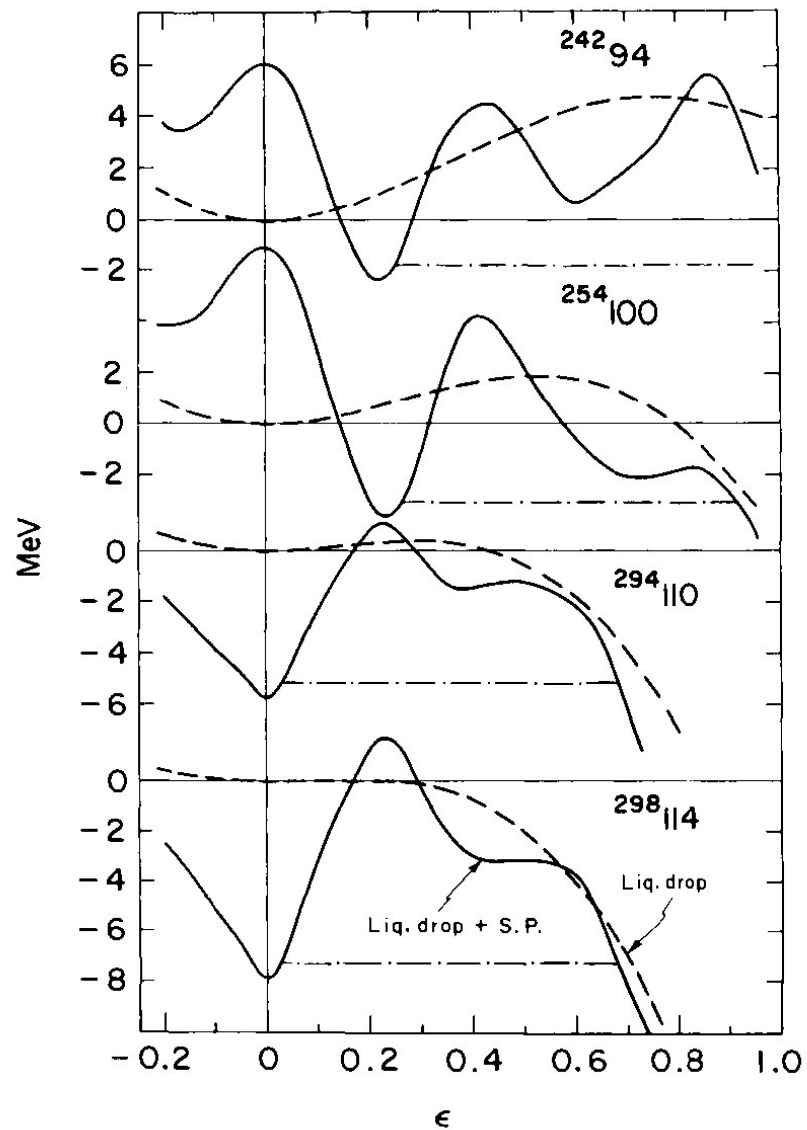
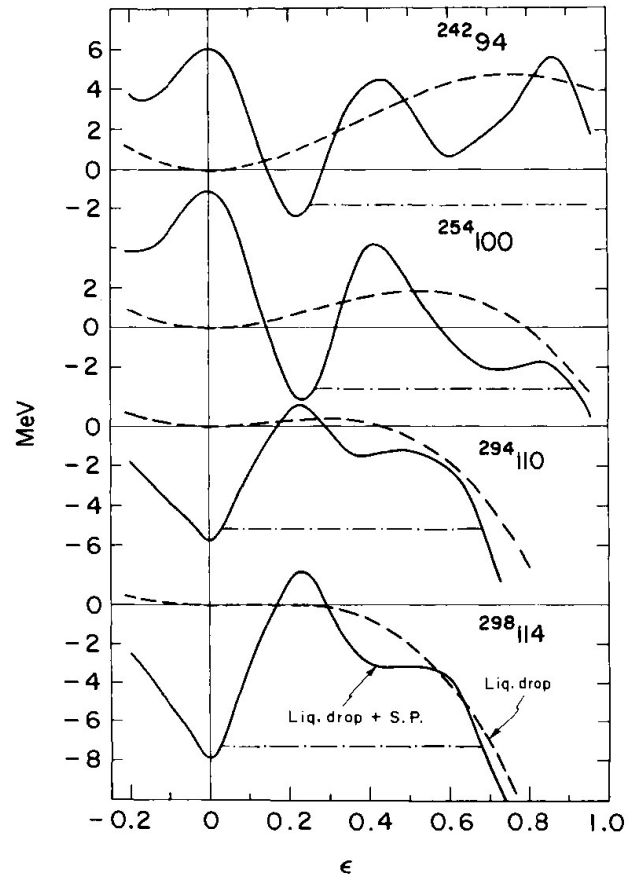


Fig. II-12. Potential energy minimized with respect to ϵ_4 as a function of ϵ for various nuclei illustrating the effect of shell structure on a liquid drop background. --- Liquid drop fission barriers; — barriers after inclusion of shell and pairing effects. [From Tsang and Nilsson (1970).]

Why (d,pf), (t,df) (t,pf) low energy reactions were very popular in 1960's and 1970's?



In many fissioning nuclei the top of the second barrier is often below the neutron threshold and one cannot “see” it in a neutron induced fission, as it would require a neutron with “negative” kinetic energy!

In the surrogate “neutron” induced fission with (d,pf), (t,df) one can probe the transition states on top of the second barrier with “neutrons with negative kinetic energy” and determine the height of the barrier.

Fig. II-12. Potential energy minimized with respect to ϵ_4 as a function of ϵ for various nuclei illustrating the effect of shell structure on a liquid drop background. --- Liquid drop fission barriers; — barriers after inclusion of shell and pairing effects. [From Tsang and Nilsson (1970).]

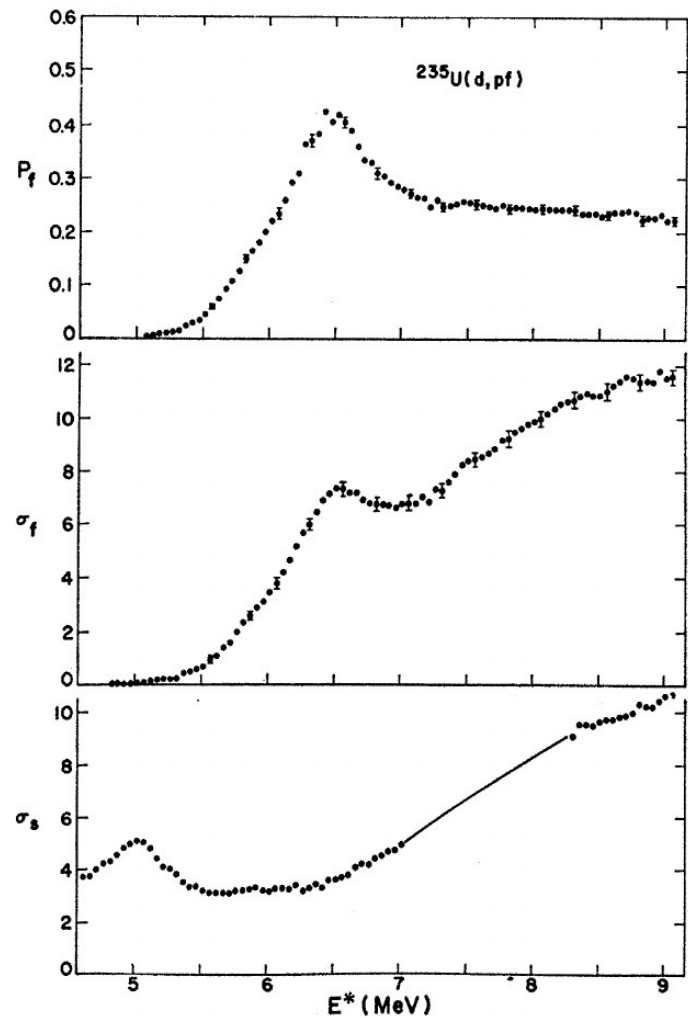


FIG. 7. Singles proton spectra (σ_s) and fission cross sections (σ_f) in arbitrary units. Resultant fission probability for the $^{235}\text{U}(d, pf)$ reaction. Solid curve in σ_s is an extrapolation underneath carbon and oxygen peaks which was used to determine P_f as described in the text.

Britt, Rickey, and Hall, PR 175, 1525 (1968)

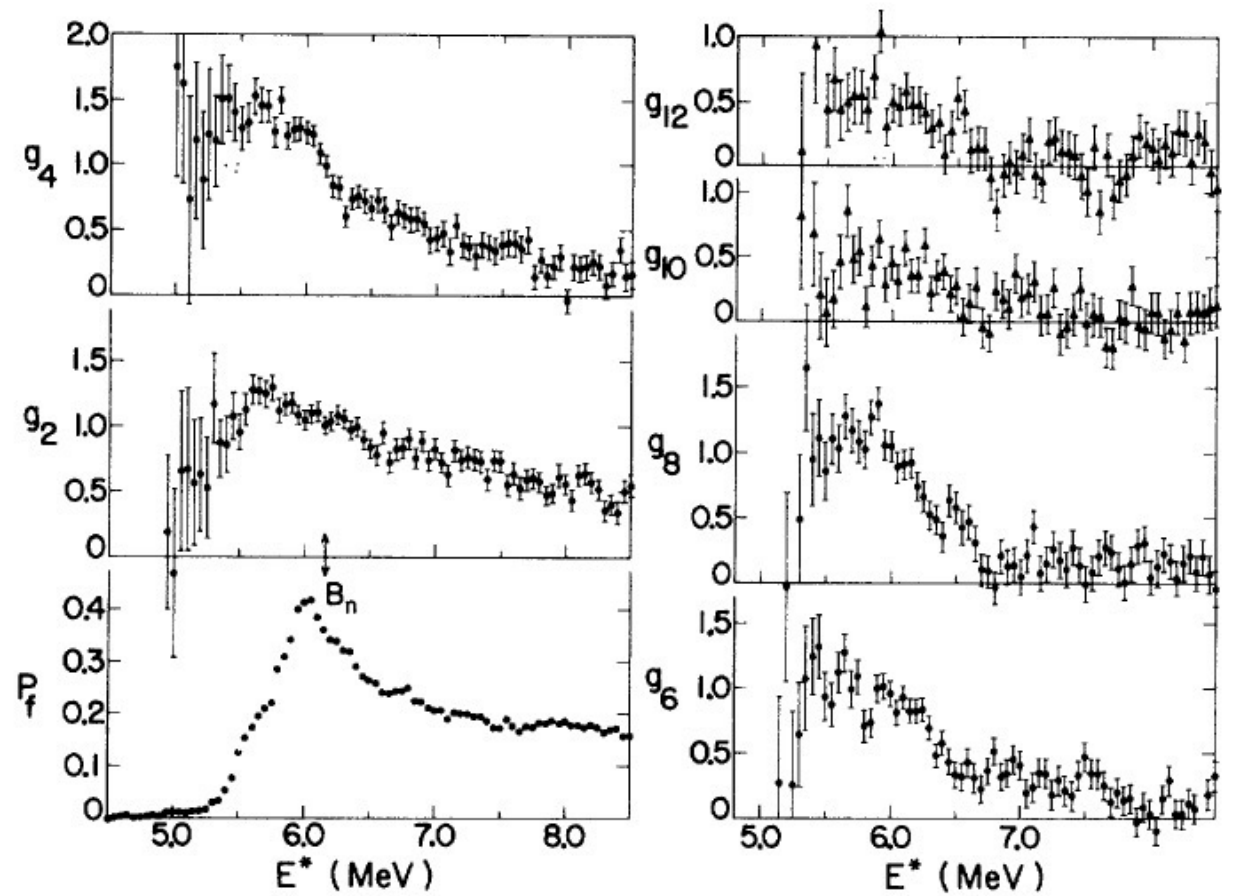


FIG. 7. Fission probability and angular-correlation coefficients for the $^{236}\text{U}(t, pf)$ reaction.

$$W(\theta) = A_0 \left[1 + \sum_{L=2,4,6,\dots} g_L P_L(\cos(\theta - \theta_0)) \right],$$

Cramer and Britt, PRC 2, 2350 (1970)

TABLE V-8

Transition nucleus	Reaction	Projectile energy	Year	Ref.
^{232}Th	$^{232}\text{Th}(\alpha, \alpha'f)$	42	1966	<i>a</i>
^{234}U	$^{233}\text{U}(d, pf)$	14.0	1959	<i>b</i>
	$^{233}\text{U}(d, pf)$	14.9	1965	<i>c</i>
	$^{233}\text{U}(d, pf)$	18.0	1968	<i>d</i>
	$^{233}\text{U}(d, pf)$	12.0	1969	<i>e</i>
	$^{233}\text{U}(d, pf)$	13.0	1969, 1970	<i>f, g</i>
	$^{233}\text{U}(t, df)$	18.0	1969	<i>h</i>
^{235}U	$^{234}\text{U}(d, pf)$	13.0	1965	<i>i</i>
	$^{234}\text{U}(d, pf)$	18.0	1970	<i>j</i>
^{236}U	$^{233}\text{U}(t, pf)$	18.0	1970	<i>j</i>
	$^{235}\text{U}(d, pf)$	14.0	1959	<i>b</i>
	$^{235}\text{U}(d, pf)$	12.5	1966	<i>k</i>
	$^{235}\text{U}(d, pf)$	10, 13, 15, 21	1967	<i>l</i>
	$^{235}\text{U}(d, pf)$	18.0	1968	<i>d</i>
	$^{235}\text{U}(d, pf)$	13.0	1969	<i>e</i>
	$^{235}\text{U}(d, pf)$	13.0	1969, 1970	<i>f, g</i>
	$^{235}\text{U}(t, df)$	18.0	1969	<i>h</i>
	$^{234}\text{U}(t, pf)$	13.0	1965	<i>m</i>
	$^{234}\text{U}(t, pf)$	18.0	1968	<i>d</i>
	$^{236}\text{U}(d, pf)$	18.0	1970	<i>j</i>
^{237}U	$^{236}\text{U}(t, df)$	18.0	1970	<i>j</i>
	$^{235}\text{U}(t, pf)$	18.0	1970	<i>j</i>
^{238}U	$^{238}\text{U}(\alpha, \alpha'f)$	43.0	1964, 1965	<i>n, o</i>
	$^{238}\text{U}(\alpha, \alpha'f)$	40.0	1966	<i>p</i>
^{239}U	$^{238}\text{U}(d, pf)$	14.0	1959	<i>b</i>
	$^{238}\text{U}(d, pf)$	18.0	1970	<i>j</i>
^{238}Np	$^{237}\text{Np}(d, pf)$	13.0	1971	<i>g</i>
^{239}Pu	$^{238}\text{Pu}(d, pf)$	13.0	1971	<i>g</i>
^{240}Pu	$^{239}\text{Pu}(d, pf)$	14.0	1959	<i>b</i>
	$^{239}\text{Pu}(d, pf)$	14.9	1965	<i>c</i>
	$^{239}\text{Pu}(d, pf)$	12.5	1966	<i>k</i>
	$^{239}\text{Pu}(d, pf)$	15.0	1968	<i>q</i>
	$^{239}\text{Pu}(d, pf)$	15.0	1968	<i>d</i>
	$^{239}\text{Pu}(d, pf)$	11.5	1969	<i>r</i>
	$^{239}\text{Pu}(d, pf)$	13.0	1969	<i>e</i>
	$^{239}\text{Pu}(d, pf)$	13.0	1969, 1970	<i>f, g</i>
	$^{239}\text{Pu}(t, df)$	18.0	1969	<i>h</i>
	$^{240}\text{Pu}(\alpha, \alpha'f)$	38.1	1966	<i>p</i>
	$^{240}\text{Pu}(\alpha, \alpha'f)$	42.0		<i>a</i>
	$^{240}\text{Pu}(p, p'f)$	20.0	1969	<i>s</i>

TABLE V-8 (continued)

Transition nucleus	Reaction	Projectile energy	Year	Ref.
^{241}Pu	$^{240}\text{Pu}(d, pf)$	18.0	1970	<i>j</i>
	$^{240}\text{Pu}(d, pf)$	13.0	1971	<i>g</i>
	$^{240}\text{Pu}(t, df)$	18.0	1970	<i>j</i>
	$^{239}\text{Pu}(t, pf)$	18.0	1970	<i>j</i>
^{242}Pu	$^{241}\text{Pu}(d, pf)$	13.0	1969	<i>e</i>
	$^{241}\text{Pu}(d, pf)$	13.0	1969, 1970	<i>f, g</i>
^{243}Pu	$^{242}\text{Pu}(d, pf)$	18.0	1970	<i>j</i>
	$^{242}\text{Pu}(t, df)$	18.0	1970	<i>j</i>
^{242}Am	$^{241}\text{Am}(d, pf)$	13.0	1971	<i>g</i>
^{244}Am	$^{243}\text{Am}(d, pf)$	13.0	1971	<i>g</i>

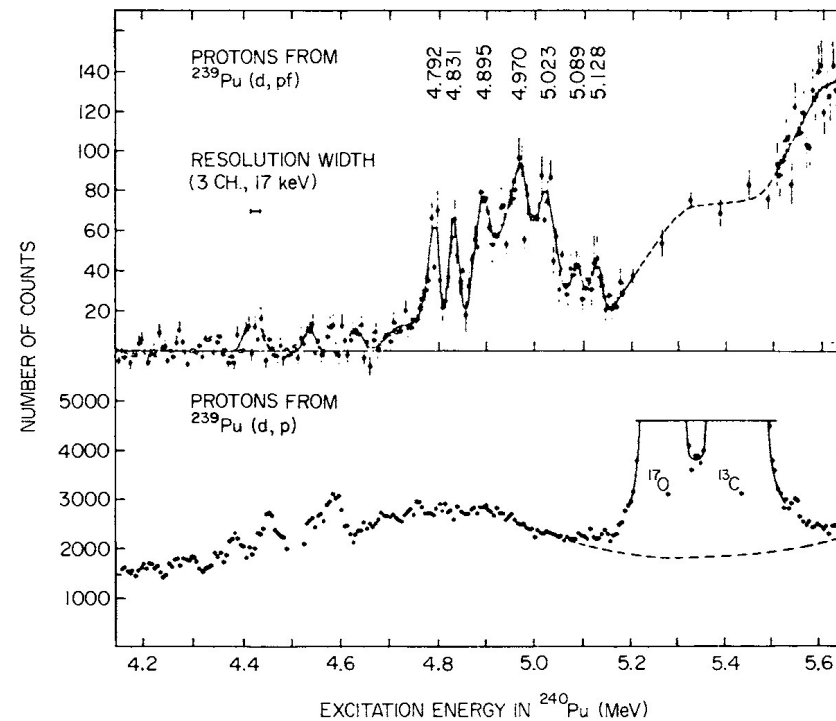


Fig. V-34. Direct experimental results showing singles and coincidence proton spectra for the $^{239}\text{Pu}(d, p)$ and $^{239}\text{Pu}(d, pf)$ reactions. Error bars give statistical deviations. [After Specht *et al.* (1969).]

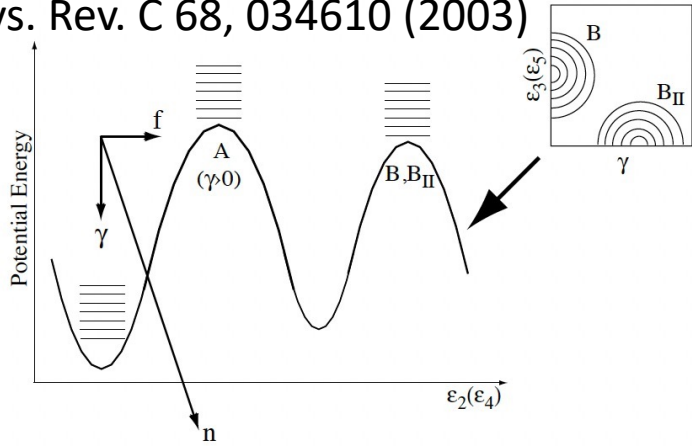


FIG. 3. Schematic representation of the statistical fission model used. The inset shows the difference between barriers B and B_{II} encountered along parallel fission paths: barrier B has a static octupole deformation, whereas barrier B_{II} is triaxial.

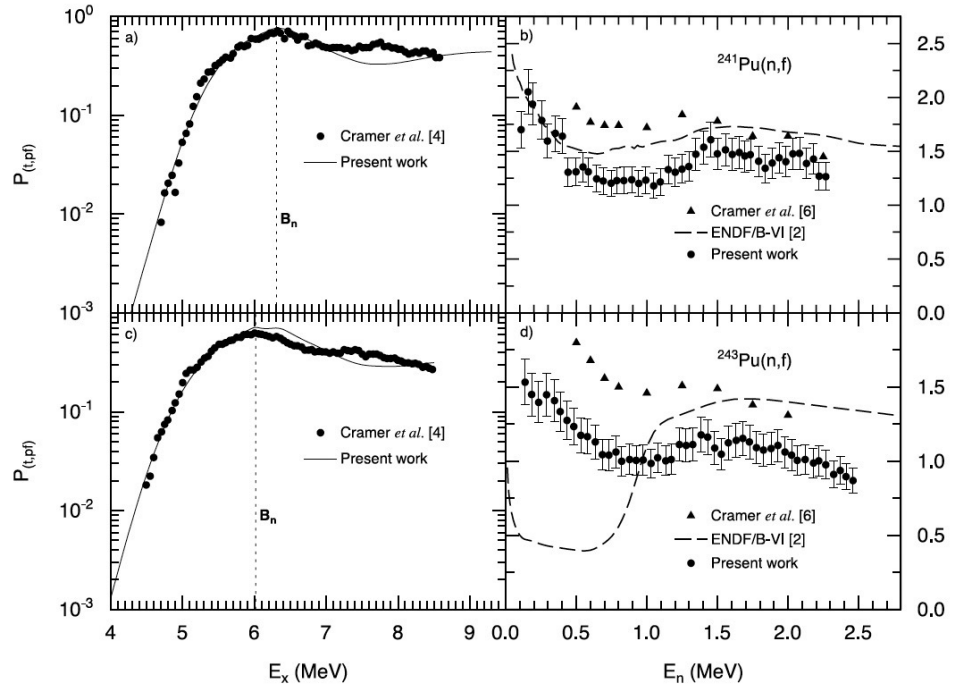


TABLE II. Comparison of average properties.

		$^{250}\text{Cf}(t, pf)$ ($4.4 < E_x < 9.8$ MeV)	$^{252}\text{Cf}(sf)$ This work	$^{252}\text{Cf}(sf)$ Ref. 13
$\overline{\text{TKE}}$	(MeV)	189.1	186.4	186.5
$\sigma(\text{TKE})$	(MeV)	14.9	12.8	12
$\langle m_L \rangle$	(u)	110.2	108.8	108.55
$\sigma(m_L)$	(u)	7.6	6.7	6.72
$\langle m_H \rangle$	(u)	141.8	143.2	143.45
$\sigma(m_H)$	(u)	7.9	6.8	6.72

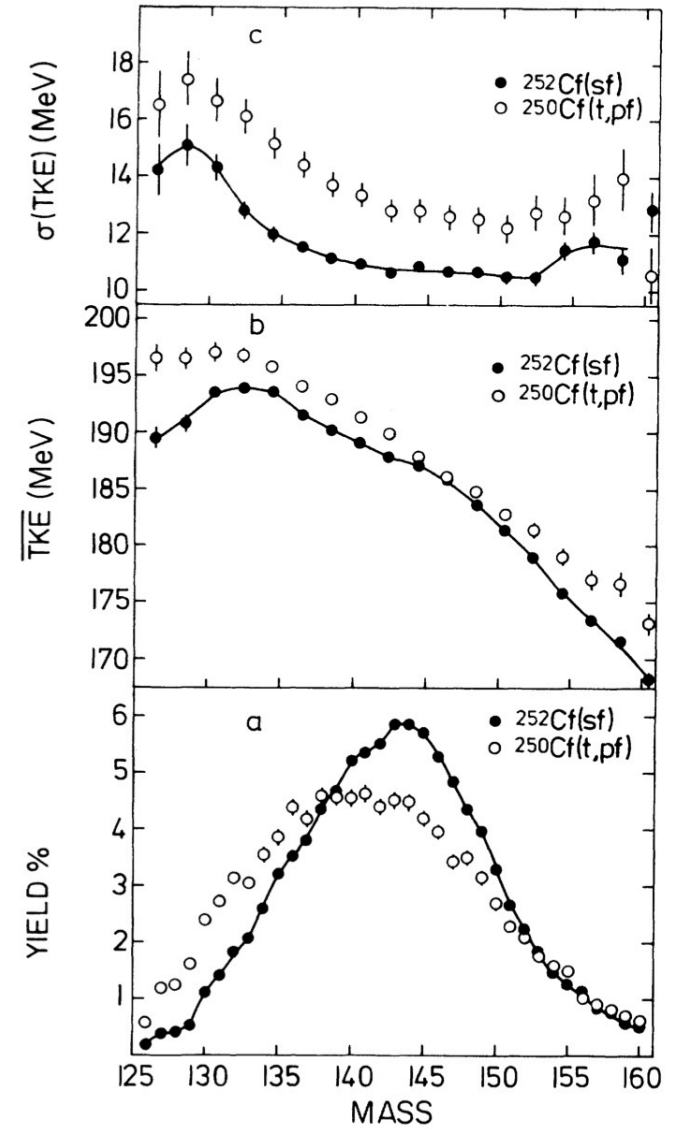


FIG. 3. Yield, average total kinetic energy, and variance of the average total kinetic energy as a function of mass for ^{252}Cf spontaneous fission and for the results of the $^{250}\text{Cf}(t, pf)$ reaction summed over the excitation energy range 5–9 MeV.

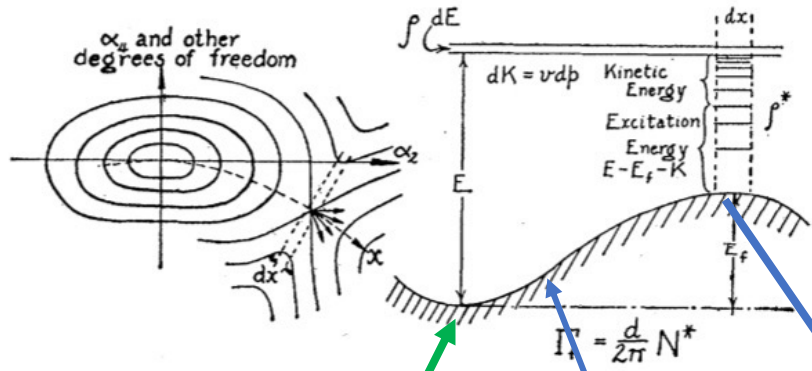
FIG. 4. Best fits (solid curves) to $P_{(t,pf)}$ data (filled circles) for (a) $^{240}\text{Pu}(t, pf)$ and (c) $^{242}\text{Pu}(t, pf)$ measurements, and the correspondingly deduced (b) $^{241}\text{Pu}(n, f)$ and (d) $^{243}\text{Pu}(n, f)$ cross sections, respectively. In panels (a) and (c), the vertical dotted line marks the position of the neutron binding energy for the compound system. Comparisons to estimated (n, f) cross sections by Cramer and Britt and to the ENDF/B-VI evaluation are also shown in panels (b) and (d).

What have we learned so far?

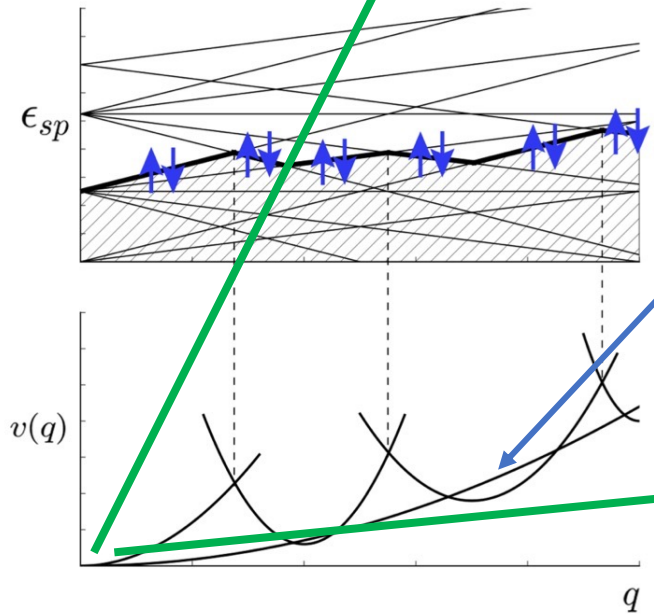
What we do not understand yet?

- One can obtain rather accurate barrier heights from surrogate reactions.
- One can extract information about the transitional states (energies, quantum numbers).
- Data extracted from different experiments: n-induced fission, (d, pf), (t, df), (t, pf) ... have great similarities, but ... there are differences, which are not yet understood.
- What are the limits of the N. Bohr (1936) compound nucleus formation assumption?

Bohr and Wheeler (1939) introduced the potential energy surface (PES) for a fissioning nucleus



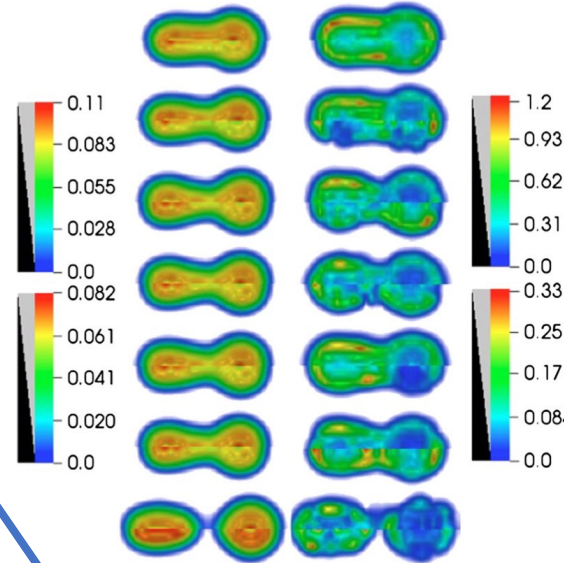
Bertsch, Phys. Lett. B, 95, 157 (1980) realized the critical role played by pairing correlations, which apparently implies adiabaticity!



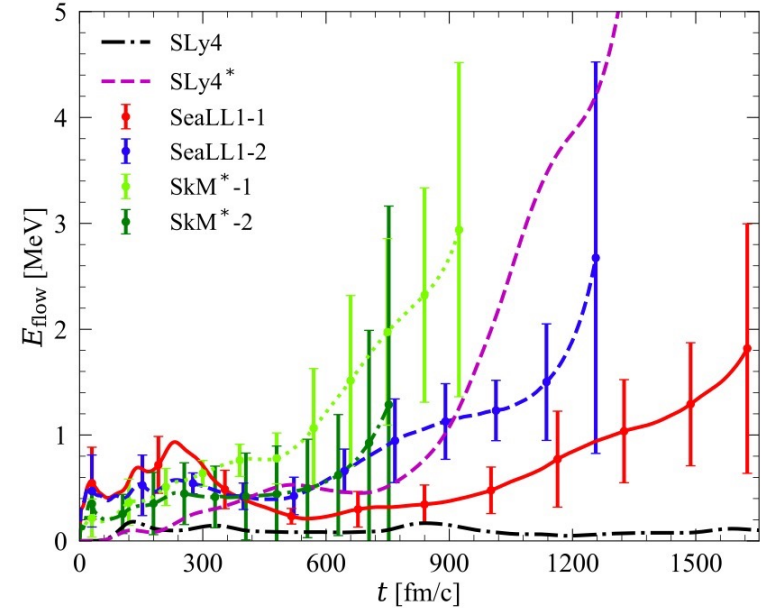
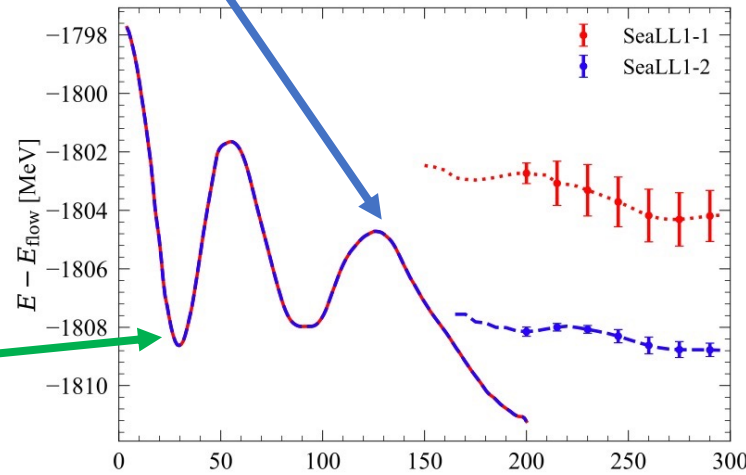
Descent from saddle-to-scission is strongly damped

Bulgac et al, Phys. Rev. Lett. 116, 122504 (2016)

Bulgac et al. Phys. Rev. C 100, 034615 (2019)



p/n densities and pairing fields



Collective kinetic energy during Saddle-to-scission descent

While on top of the saddle configuration the nuclear level density is relatively small $O(1)/\text{MeV}$, see A. Bohr (1956), at scission the level density is of order $O(10^5) \text{ MeV}^{-1}$ and the single PES should be replaced with $O(10^5)$ PESs, as adiabaticity is strongly violated, similarly to the dynamics of molecular systems discussed for many decades, and level density is $O(1-10) \text{ eV}^{-1}$, see Bulgac et al. *Front. Phys.* 8, 63 (2020), Bender et al, *J. Phys. G*, 47, 113002 (2020).

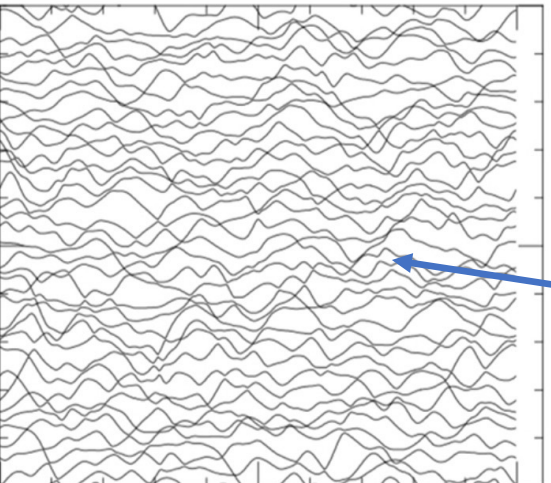
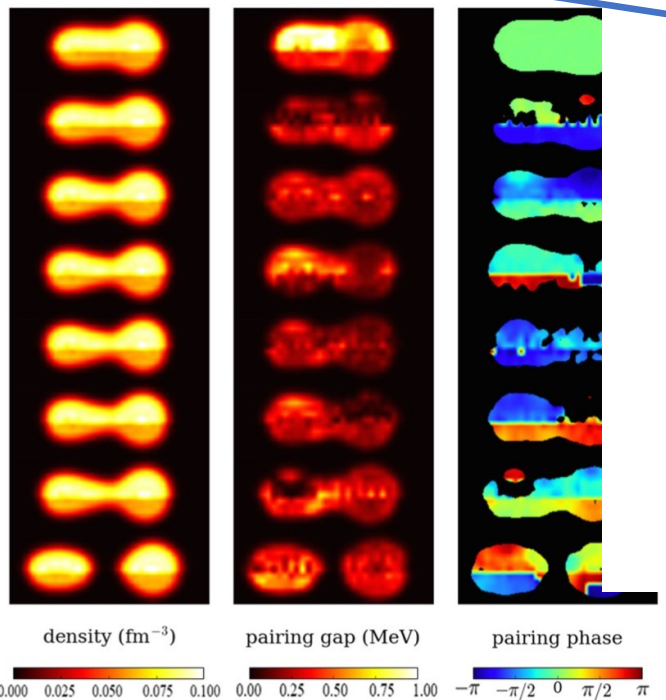


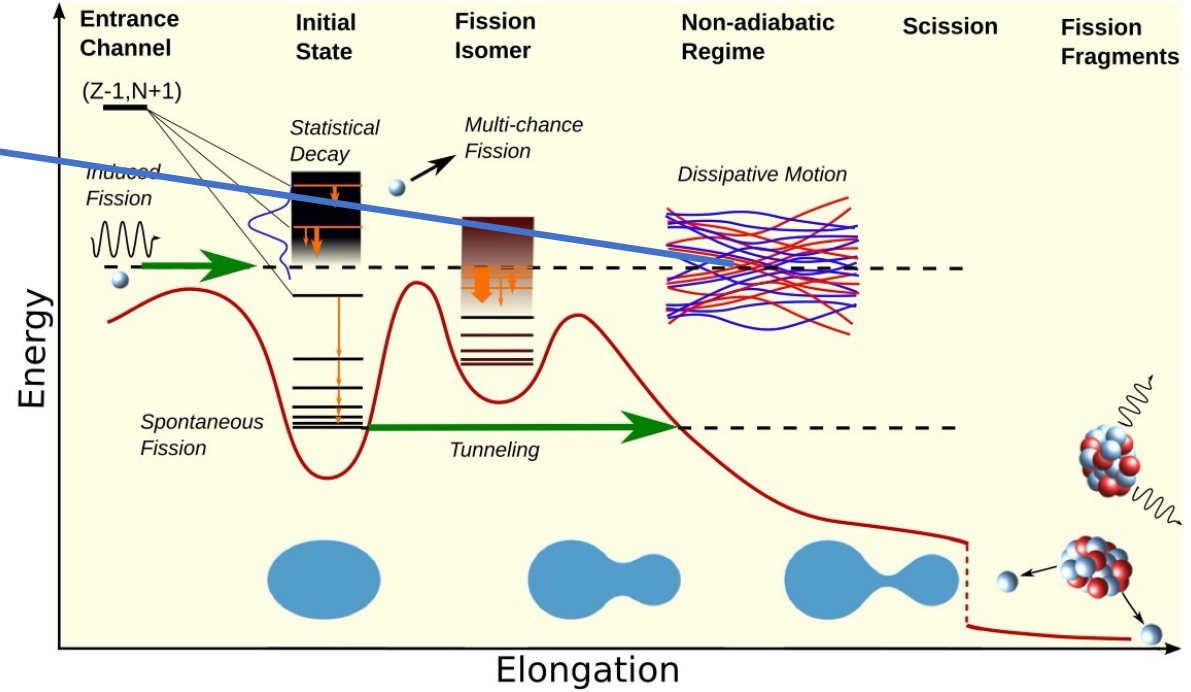
FIGURE 3. In nuclei the level density increases with the excitation energy quite fast, practically exponentially at energies of the order of the neutron separation energy, when $\rho(E^*) \propto \exp(\sqrt{2aE^*})$ [57, 58], and it reaches values of $O(10^5) \text{ MeV}^{-1}$ and various potential energy surfaces, corresponding to different "molecular terms" display a number of avoided level crossings. Here we illustrate the generic behavior of the collective energy levels (y-axis) as a function of collective coordinate (x-axis), see Bulgac et al. [59] for details in a similar figure.

²⁴⁰Pu fission with the normal pairing gap



Pairing correlations survive even at scission, where the nuclear temperature is of order 1 MeV

Bulgac, *Phys. Rev. C* 100, 034615 (2019)



Present theoretical findings into the non-equilibrium fission dynamics (seven parameters and no phenomenology):

- Achieved scission and full separation of fission fragments in a pure microscopic framework starting near the outer fission barrier, without any assumptions.
- Established the strong damped character of the large amplitude collective motion beyond the outer saddle-point.
- Fission fragments excitation energies and their sharing mechanism before and after they are fully separated (TXE).
- Strongly damped character of fission fragment shape evolution after they are fully separated.
- Total kinetic energy of fission fragments (TKE).
- Evolution of these properties with the initial excitation energy of the compound nucleus.
- Evaluated the intrinsic fission fragments spins and their correlations.
- Properties of neutrons emitted before fission fragments are fully accelerated.

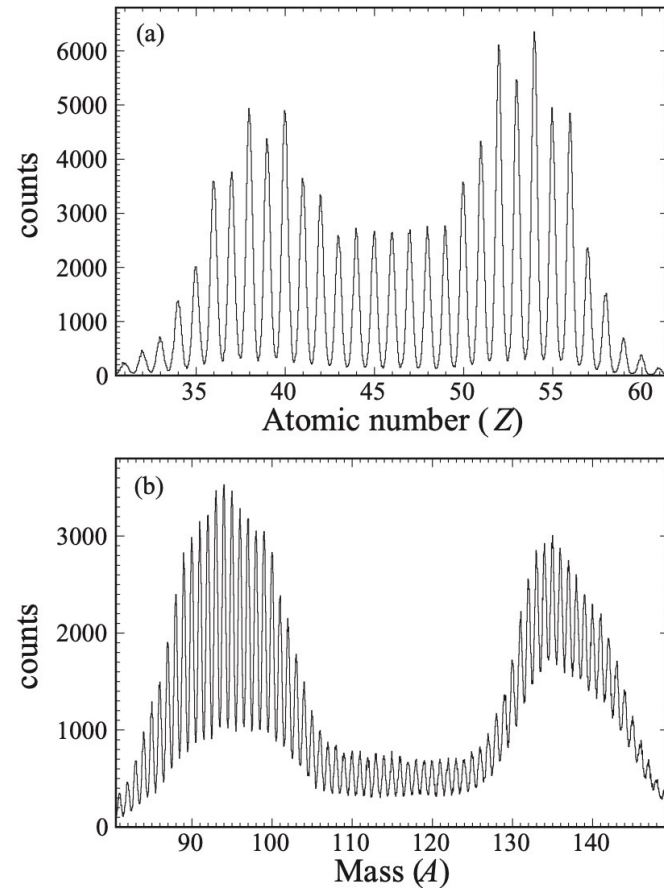


FIG. 4. Measured atomic-number distribution (a) and mass distribution (b) of the fission fragments produced by Coulomb-induced fission of ^{235}U . For details see text.

Experimentally, one can achieve quite accurate FF charges and masses.

Measuring the FFs TKE, TXE, their masses and charges as a function of excitation energy of the compound fission nucleus and comparing the data to the most microscopically founded theory will lead to a deeper understanding of this complex quantum non-equilibrium process.

Further theoretical developments required:

- Inclusion of quantum fluctuations.

In the time-dependent density functional theory (TDDFT) extended to superfluid systems one can study presently only separate “classical trajectories” of the nuclear system. What is missing is the interference between different trajectories, as in the case of the two-slit experiment.

The major difference with the two-slit experiment is that each interfering “trajectory” has also an internal structure and in this case, surprisingly, interference is really happening. (New results are expected soon!)

- The theoretical formalism for even-even nuclei and odd mass and odd-odd nuclei has qualitative differences, since in systems with odd number of fermions time-reversal symmetry is spontaneously broken and new qualitative terms appear in the density functionals, over which we do not have a very good control and sufficient knowledge yet. (New results are coming out soon!)
- The fission of even-even, odd mass and odd-odd nuclei show significant qualitative differences, see Vandenbosch and Huizenga, Nuclear Fission (1973), which (partially) can be attributed to pairing correlations.

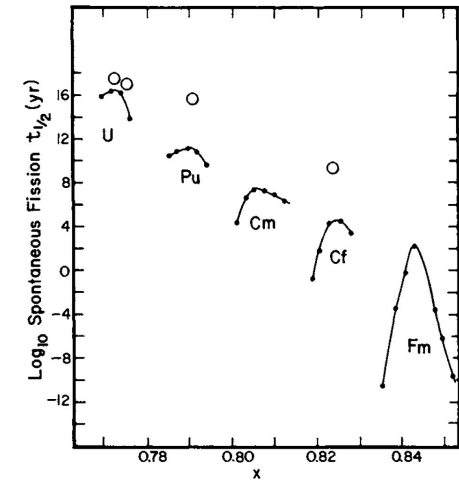


Fig. III-1. Spontaneous fission half lives of even-even (●) and even-odd (○) nuclides as a function of the fissility parameter x appropriate to the Myers-Swiatecki (1967) mass formula. (In this formula x is not simply proportional to Z^2/A , since there is a composition-dependent correction term proportional to $[(N-Z)/A]^2$ in the surface energy expression.)

Why a (t, pf) reaction would be a great tool to study fission?

Q – value $[(A,Z)+t \rightarrow (A+2,Z) + p] \approx 2.25 - 5.67 \text{ MeV}$ (Apart from Aage Bohr (1956) arguments.)

- $\Delta J^\pi = 0^+$, $\Delta(N - Z) = 2$. Consequently, the same quantum numbers as for a neutron Cooper pair and spectrum of excited states in the compound nucleus states are simpler in $^{234}\text{U}(t, pf)$ than in $^{235}\text{U}(n,f)$ for example. The presence of the pairing condensate in the even-even nucleus would enhance the pair transfer and introduce hopefully the least “disturbance” of the structure of the target nucleus, unlike in odd-mass nuclei, where one accesses higher spins compound nuclear states.
 $^{233}\text{U} (5/2+)$, $^{235}\text{U} (7/2-)$, $^{237}\text{U} (1/2+)$, $^{239}\text{U} (5/2+)$,
 $^{237}\text{Pu} (7/2-)$, $^{239}\text{Pu} (1/2+)$, $^{241}\text{Pu} (5/2+)$, $^{243}\text{Pu} (7/2+)$, $^{245}\text{Pu} (9/2-)$
- One can control the excitation energy spectrum of the compound nucleus, starting below the neutron threshold.
- As recently observed by Britt et al. in several studies the reactions (t, pf), (sf), (n,f), etc. show some differences, and other similar reactions show differences, unexpected if the N. Bohr (1936) assumption of the formation of a compound nucleus is valid in these reactions.

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Weber, Britt, Wilhelmy
 Phys. Rev. C 23, 2103 (1981)

Younes and Britt,
 Phys. Rev. C 68, 034610 (2003)

